# Alice in Randomland: A mathematical adventure

Camila, Brito P.
Center for Mathematical Modeling, Universidad de Chile, Santiago, Chile
*camila.brito.p@gmail.com*

Salomé, Martínez S.
Center for Mathematical Modeling, Universidad de Chile, Santiago, Chile
*samartin@dim.uchile.cl*

Grecia, Gálvez P.
Center for Mathematical Modeling, Universidad de Chile, Santiago, Chile
*grecia.galvez@gmail.com*

Flavio, Guiñez A.
Center for Mathematical Modeling, Universidad de Chile, Santiago, Chile
*flavioguinez@gmail.com*

Thomas, Peet M.
Center for Mathematical Modeling, Universidad de Chile, Santiago, Chile
*tomaspemora@gmail.com*

Ricardo, Salinas P.
Center for Mathematical Modeling, Universidad de Chile, Santiago, Chile
*ricardosalinaspaez@gmail.com*

## Abstract

*The different nature of probabilistic thinking with respect to other areas of mathematics, coupled with traditional and rigid teaching models, makes the teaching of probability a real challenge for teachers. This article describes the design, characteristic and the underlying mathematical ideas of Alice in Randomland, which is an interactive mathematical story that promotes mathematics learning through a problem-solving approach, in which illustrations aid the visualization of math concepts and strategies. The storybook is complemented by an app for Smartphones that contains interactive mobile games to help children simulate experiments, explore and analyze problems, enhancing a visual discovery of the math ideas appearing throughout the story.*

Keywords: math education, mathematical story, learning resources, probability approaches, interactive story, mobile app, random games.

## Introduction

In Chile, as in the rest of the world, mathematics education is currently conceived as a space to develop key competencies for modern life. This is evident in the conception of mathematical literacy defined in PISA (OECD, 2016). In school education there have been changes of school curricula moving from an organization centered mainly on content, to explicitly considering the development of mathematical skills and the learning of mathematics in context. The Chilean school curriculum (MINEDUC, 2012) aims for students to develop mathematical thinking, which defines as a capacity that allows to understand the relationships that occur in the environment, quantify them, reason about them, represent them and communicate them. In this context, children, from an early age, should have opportunities to reason mathematically, solve problems, communicate findings, use abstract and precise language, visualize objects and relationships, understand abstract concepts, develop operative skills, among others. These ambitious goals imply a huge challenge for all teachers.

Also, in recent years the mathematical knowledge that is addressed in school mathematics has expanded. The importance of understanding and analyzing phenomena that involve uncertainty, and the need to manage and give meaning to an increasing amount of data, has determined that statistics and probabilities are incorporated as a longitudinal strand of school mathematics from primary education. In

Chile, the presence of this content strand dates back approximately to the year 2000, starting in the 1st grade the subjects related to statistics (data), and in 5th grade those related to probability. The teaching of probabilities and statistics presents great and unique challenges. For example, in Chile less than half of the pre-service primary school teaching programs do not include topics of probability in their basic training (MINEDUC, 2016). Moreover, as indicated in Batanero (2015), even if in their initial training teachers that have a major in mathematics, they may be unfamiliar with different meanings of randomness and probability, or with their students' most common misconceptions. On the other hand, the teaching of this topic presents its own challenges. As mentioned in Batanero & Díaz (2012), teaching principles valid for other areas of mathematics, are not always appropriate in the field of probability and this is something teachers are not necessarily aware of.

In this scenario, teaching must go beyond the workbooks and textbooks and teachers should teach math in creative, relevant, and meaningful ways (Jennings et al., 1992). Therefore, it is necessary to create learning resources coherent with the current conception of the teaching of mathematics that provides opportunities for inquiry and the development of mathematical reasoning and that captures the interest of students. Mathematical stories provide a way to present mathematics in meaningful and imaginative contexts. As suggested in Nicol and Crespo (2005), imaginative tasks offer opportunities for a kind of intellectual and emotional engagement often missing from school mathematics tasks.

In this work, we describe the elaboration and characteristics of Alice in Randomland, an interactive mathematical story aimed at 11-15 years old students, inspired in characters and elements from Lewis Carroll's classic to stimulate interest for mathematics and provide opportunities for math learning. This book tells the story of Alice, on her first day at school she finds herself in a journey of adventures through a magical land where games of chance are the main activity of its inhabitants. To overcome the challenges of this world, our heroine is forced to think about the situations of uncertainty, leading her to discover fundamental ideas on probability, that she will use to solve and analyze problems. The book promotes mathematics learning, through a problem-solving approach, facilitated by interactive mobile games for Smartphones that help children to explore problems involving probabilities and lead to a visual discovery of math ideas appearing throughout the story.

## Framework: Teaching and learning probability in elementary school

Batanero, Godino and Roa (2004) discussed some of the unique challenges involved in the teaching of probability. In this article, they state that probabilistic reasoning is different from logical or causal reasoning and so counterintuitive results can be found in probability even at very elementary levels, which contrast with other branches of mathematics, where these types of results are encountered only when working at a high degree of abstraction (Borovcnik and Peard; 1996). This distinctive feature of probability may explain the existence of misconceptions and learning difficulties that remain in high school level. To face learning difficulties is important to provide opportunities for students to confront their own misconceptions and erroneous beliefs (Batanero, 2004). As discussed by Borovcnik and Kapadia (2014b), the development of mathematical concepts is usually accompanied by ruptures and conflicts, and there is an opportunity for learning when one tries to solve the conflict and understand paradoxical results.

Approaches regarding the understanding of randomness and probability, which are consistent with the Chilean curriculum are discussed in Batanero (2015). According to this paper, *in primary school, children should be encouraging to discriminate certain, possible and impossible events in different contexts, and use the language of chance. For these specific materials with symmetrical properties, such as dice or coins should be used, so that children can compare their predictions from the a-priori analysis of the structure with frequency from data collected from repeated experiments to estimate probability*. It is also proposed that materials lacking symmetry properties are introduced in a second stage, for which probability can only be initially estimated from frequencies. Starting in early middle school (10-11-year-old) children can start simulating simple situations using devices. As stated in this paper, although simulation or experimentation with dice and coins have an important function in stabilizing children's intuition and in materializing probabilistic problems, they do not provide the key to how and why the problems are solved. Thus, it is necessary to introduce combinatorial schemes or tools like tree diagrams for children to start to understand the solution of probabilistic problems. This highlights the complementary nature of classical and frequentist approaches to probability (Batanero, 2015).

## Description of the math story and key features.

Alice in Randomland was created in a mathematics research center by a multidisciplinary team composed by mathematicians, math teachers, engineers, as well as designers, illustrators, screenwriters and programmers. The story was created with the aim to motivate children to learn mathematics and presenting some key ideas about probability. This was done through a narrative combining mathematical riddles with a world of fantasy, inspired by the classic book of Lewis Carroll. The story is a *genuine mathematical story* (Borasi, Sheedy & Siegel; 1990), that is mathematical ideas are key for the setting, characterization and plot.

The illustrations in the book not only serve to give life to the different characters and magical landscapes, but are also used to support the visualization and comprehension of mathematical ideas, and as a tool to represent mathematical strategies and procedures to solve problems. Given that representation is considered a key mathematical skill (OECD; 2016), the illustrations in the story have the potential to enhance significantly its usefulness as a teaching and learning device.

The story was divided by chapters, in each one the protagonist had to overcome a challenge which involve playing and analyzing a game of chance. Each chapter was centered around an important idea of probability, and the sequence was designed to promote understanding. The description of this sequence with some examples will be discussed below.

The book includes a mobile app conceived to learn by playing. By using it, the reader can experiment and analyze situations described in the story, accessing interactive games through QR codes found all over the book. The apps are not necessary to follow the storyline, they are conceived to aid the reader to deepen the exploration of the presented problems, as well as to extend and apply strategies to solve related problems.

## The story in connection to a learning sequence

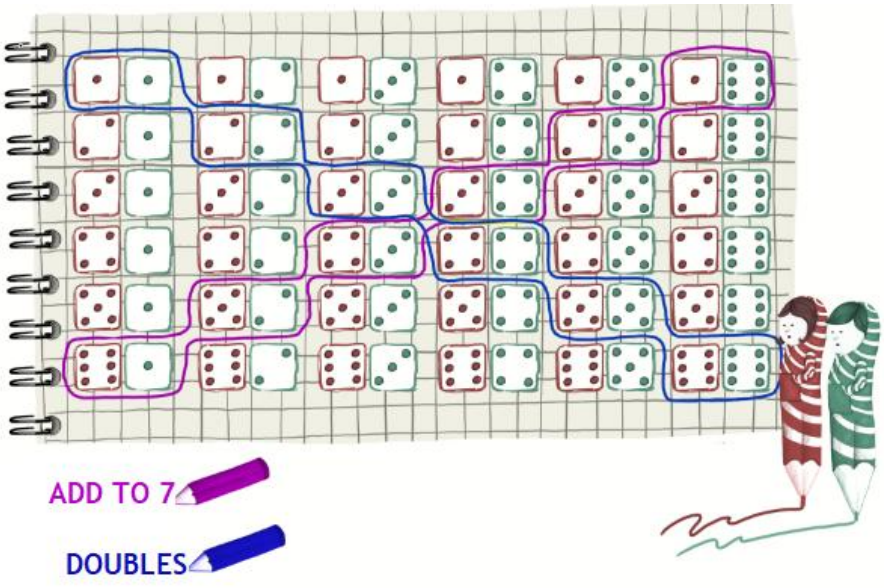
In the first chapters of this book random experiments, such as games with dice and cards, are presented, and they are analyzed through the frequentist or empirical notion of probability. In one of them, Alice, the protagonist, is confronted with the game of cards, in which she wins if she takes out two equal cards of a hat containing two red cards and a blue one (as in Figure 1). In this non-symmetrical game, an erroneous belief becomes evident: if an experiment has two possible outcomes, both have the same probability. In the story, the characters repeat the game many times and one of them says that he knows what will eventually happen "because of his experience of having played the game many times". For this game, a mobile application that includes a simulation of the games is included, giving the reader the opportunity to dynamically view the results obtained by playing the game a lot of times, and thus contrast their own beliefs with the empirical results obtained.

Once the protagonist has had her first experiences with games of chance, she is presented with new challenges that involve analyzing the results of a random experiment without simulating it. For this, Alicia will resort to representations and heuristics that facilitate the search and analysis of possible cases of a randomized experiment. In this way, the reader grasps the classical notion of probability through problems that involve combinatorics and that require the development of elementary counting strategies. For instance, in chapter 3 a dice game is introduced that involves analyzing the occurrence of events of equal probability, motivating the use of a classical approach of probability in which all possible cases must be found (this is shown in Figure 2). This game is extended in its associated mobile application, allowing the reader not only to simulate the results of the new game, but to analyze it through representations that allow executing counting strategies.

In the last chapters, the complementary nature of empirical and classical approaches is emphasized. On the one hand, there are situations where the reader can understand how the analysis of possible cases allows to explain empirical results of games with dice and cards, which can be counterintuitive. On the other hand, the protagonist is confronted with situations where she uses her knowledge from the repetition of an experiment and complements it with the analysis of possible cases. With this, she is able to anticipate the outcome of a game, when the underlying random experiment is repeated many times.

Figure 1 - Game of cards

Figure 2 - Dice game



## References


Batanero, C. (2015, February). Understanding randomness: Challenges for research and teaching. In CERME 9-Ninth Congress of the European Society for Research in Mathematics Education (pp. 34-49).

Batanero, C., & Díaz, C. (2012). Training teachers to teach probability: Reflections and challenges. Chilean Journal of Statistics, 3(1), 3-13.

Batanero, C., Godino, J. D., & Roa, R. (2004). Training teachers to teach probability. *Journal of statistics Education*, *12*(1).

Batanero, C., & Sanchez, E. (2005). What is the Nature of High School Students' Conceptions and Misconceptions About Probability?. In *Exploring probability in school* (pp. 241-266). Springer, Boston, MA.

Borasi, R., Sheedy, J. R., & Siegel, M. (1990). The power of stories in learning mathematics. *Language Arts*, *67*(2), 174-189.

Borovcnik, M., & Kapadia, R. (2014b). From puzzles and paradoxes to concepts in probability. In E. J. Chernoff & B. Sriraman, (Eds.), Probabilistic thinking: presenting plural perspectives (pp. 35-73). Dordrecht: Springer.

Borovcnik, M., and Peard, R. (1996), “Probability”, in International handbook of mathematics education, eds. A. Bishop, et al., Dordrecht: Kluwer, 239-288.

Jennings, C. M., Jennings, J. E., Richey, J., & Dixon-Krauss, L. (1992). Increasing interest and achievement in mathematics through children's literature. Early Childhood Research Quarterly, 7(2), 263-276.

MINEDUC (2012) Bases Curriculares Educación Básica. 1º edición, Santiago, Chile.

MINEDUC (2016) Technical Report. Santiago. Chile

Nicol, C., & Crespo, S. (2005). Exploring mathematics in imaginative places: Rethinking what counts as meaningful contexts for learning mathematics. School Science and Mathematics, 105(5), 240-251.

OECD (2016). *PISA 2015 assessment and analytical framework: Science, reading, mathematics and financial literacy*. OECD publishing.